Thus using an isotropic mean free path Λ we calculated μ_{11} for 1, 2, and 4 lower valleys as a function of N following Csavinszky's treatment[13] of the partial-wave analysis but including the dependence of screening on the number of valleys as expressed in Eq. (2). The results are shown together with the experimental curves in Fig. 7. The same figure also shows calculations based on the less accurate Born approximation and on the above assumptions.

The experimental mobilities are considerably higher and decrease less rapidly with increasing N than those of the partial-wave treatment. This probably has to be attributed to the failure of the individual scattering assumption. In contrast to previous experience[26] the Born approximation yields a higher mobility than the partial wave treatment. Both theories predict $\mu_{11}(4) > \mu_{11}(2)$ and $\mu_{11}(1)$ because of the more efficient screening when the electrons are in 4 valleys. The effect, however, is greatly reduced by the decrease of the scattering cross section with increasing E_F. The calculated mobilities $\mu_{11}(1)$, $\mu_{11}(2)$, and $\mu_{11}(4)$ decrease with increasing N. The slope increases with the number of lower valleys. This causes $\mu_{11}(1)$ to become larger than $\mu_{11}(2)$ at some concentration and to become even larger than $\mu_{11}(4)$ at very high concentrations.

The experimental mobility curves decrease less fast than both sets of theoretical curves. This discrepancy as well as the fact that the partial-wave treatment of Csavinszky overestimates the scattering probably result from the failure of the individual scattering model.

The magnitudes of the experimental $\mu_{11}(2)$ and $\mu_{11}(4)$ of Fig. 7 depend on the choice of the mobility anisotropy ratio K in Eq. (1). If K is not constant as we assumed but increases to $K=4.4$ with the number of lower valleys then cases for which $\mu_{11}(2)<\mu_{11}(1)$ can be obtained.

C. Low-Stress Piezoresistance

Considering only the resistivity change resulting from the shear-induced shifts of the [111] conduction-band valleys,[27] one obtains for the piezoresistance coefficient $\Pi=\Delta\rho/\rho X$ in degenerate n-type germanium for the orientations denoted by F and C, respectively,

$$\Pi(F)=-\frac{2}{9}\frac{K-1}{2K+1}(\tfrac{3}{2}+s)\frac{E_2 S_{44}}{E_F},\qquad \Pi(C)=\tfrac{3}{4}\Pi(F). \tag{4}$$

Here s is the power of the explicit energy dependence of the mobility which is assumed to have the form

$$\mu=\text{const}N^r E^s. \tag{5}$$

The factor N^r expresses the explicit concentration dependence of μ. The assumption of a constant stress-independent K implies that r and s are the same for μ_{11} and $\mu_\perp$. E_F in Eq. (4) is the zero-stress Fermi energy.

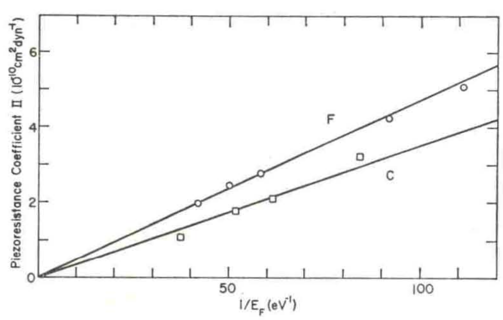

FIG. 8. The low-stress longitudinal piezoresistance coefficient as a function of reciprocal Fermi energy for [110] uniaxial compression (C) and [111] uniaxial compression (F).

The experimental values of $\Pi(F)$ and $\Pi(C)$ are plotted against $1/E_F$ in Fig. 8. From the slope of either one of these curves and the value $K=3.9\pm0.1$ the quantity s can be determined. We find $s=0.79\pm0.03$. A slightly larger value of $K=4.3$ would yield $s=0.7\pm0.03$. At low stresses the four valleys remain nearly degenerate. Since the value of s might depend on the number of lower valleys because of the change in screening, the value thus determined is $s(4)$. The slopes of the mobility curves in Fig. 7 yield the quantity $r+2s/3$ because the mobilities are determined at $E=E_F$ and $E_F\propto N^{2/3}$. With the value $s(4)=0.79$ and the slope of the $\mu(4)$ versus N curve we find $r(4)=-0.72$.

We now compare these values with those predicted by Csavinszky's theory. The theory yields an isotropic scattering cross section and hence assumes K to be stress-independent in agreement with our analysis.

The form of the mobility as expressed in Eq. (5) is fairly well justified in the concentration range $5\times10^{18}\leq N\leq10^{19}$ cm^{-3} of interest here. For the four-valley case the theory predicts $s(4)=0.3$ and $r(4)=-0.71$. A comparison with the experimentally determined values $s(4)=0.79$ and $r(4)=-0.72$ shows a rather large disagreement of the s values and a surprisingly good agreement of the r values. One would expect the failure of the individual scattering hypothesis to have a much stronger effect on r. Even in the individual scattering theory r deviates from $r=-1.0$ because of the concentration-dependent screening effects. These effects become apparent when one compares the values of r and s for different numbers of lower valleys. The theory yields $s(2)=0.52$, $r(2)=-0.82$, and $s(1)=0.59$, $r(1)=-0.85$ in the same concentration range.

Because of the close agreement of the r values we believe that the failure of the individual scattering

[26] J. M. Ziman, *Electrons and Phonons* (Clarendon Press, Oxford, England, 1962), p. 342; F. J. Blatt, Phys. Chem. Solids **1**, 262 (1957).

[27] The magnitude of effects on the piezoresistance which do not originate from a shear-induced shift of the valleys was determined by measuring the change in resistivity under [100] compression. The piezoresistance coefficient found was to be smaller than 3% of the smallest Π obtained in arrangement C. The corrections due to the stress-induced geometry change is always less than 1%. These minor corrections have been neglected in the analysis.

hypothesis is not the only cause for the disagreement between theory and experiment. In determining the s value from the Π versus E_F^{-1} curves of Fig. 8 we used the value of the mobility anisotropy K which was directly obtained from a measurement of $\mu_{\parallel}(1)$ and $\mu_{\perp}(1)$. If K changes as the electrons are transferred from four valleys to one valley, then such a simple analysis of the experimental data is impossible.

IV. CONCLUSIONS AND SUMMARY

The mobility of Sb-doped Ge at 1.2°K increases with concentration above as well as below the critical concentration N_c at which the transition from nonmetallic to metallic conduction occurs. No change in slope marks the occurrence of this transition on the μ versus N curves. This indicates that even above N_c the mobility is governed by the exchange interaction of the randomly distributed impurities.

Only at much higher impurity concentrations, $N \geq 10^{18}$ cm^{-3} for zero stress and $N \geq 3\times 10^{18}$ cm^{-3} for large [111] compressions, does the mobility decrease with increasing N and thus show the behavior expected from a metal under residual resistance conditions.

Hence there seem to exist in germanium at very low temperatures the following concentration ranges with different conduction processes:

(a) $N < 10^{16}$ cm^{-3}, impurity or hopping conduction;

(b) $3\times 10^{16} < N < 10^{17}$ cm^{-3}, hopping conduction or impurity band conduction with the existence of a thermal activation energy ϵ_2;

(c) $10^{17} < N < 10^{18}$ cm^{-3}, impurity band conduction with $\epsilon_2 = 0$; and

(d) $N > 10^{18}$ cm^{-3}, metallic conduction similar to a metal in its residual resistance range.

The concentrations limiting these ranges are usually not sharply defined. Furthermore, they depend on the doping element and on the state of stress in the semiconductor.

The mobility anisotropy $K = \mu_{\perp}/\mu_{\parallel} = 3.9 \pm 0.1$ was obtained directly by measuring the resistivity component perpendicular and parallel to the valley axis at large [111] compressional stresses in the high-concentration range $4\times 10^{18} \leq N \leq 9\times 10^{18}$ cm^{-3}. This value is close to $K = 4.4$, the theoretical value obtained with an isotropic mean free path, and a mass anisotropy equal to that of pure germanium.

We obtained the explicit energy and concentration dependence of the mobility for the four-valley case assuming the mobility anisotropy K to be stress independent. Csavinsky's calculations overestimate the magnitude of ionized impurity scattering by approximately a factor of 2 and also the total dependence of the mobility on the number of scattering centers. Both of these effects indicate that the electrons do not experience scattering at independent donor ions but rather are influenced by the potentials of several ions. The analysis also indicates that the mobility anisotropy may change with the distribution of the electrons over the various valleys.

It would be interesting to compare these results on Sb-doped Ge with similar measurements[6] on As-doped Ge. It is known[28] that the mobilities of unstressed Ge doped with As are lower by a factor of 1.4 than those of Ge doped with Sb. This is attributed[29] to the large central cell potential of the As impurities which affects the intravalley scattering and which might lead to an appreciable intervalley scattering. At large [111] compressions we find, however, that the ratio $\mu_{\parallel}(\mathrm{Sb})/\mu_{\parallel}(\mathrm{As})$ is approximately 2. The fact that this ratio is larger in the one-valley case than in the four-valley case indicates that intervalley scattering does not seem to be the dominant cause for the difference in mobilities observed with different donor elements. A quantitative analysis of the measurements on As-doped Ge is complicated by the fact that the piezoresistance continues to decrease even at the highest compressional stresses and fails to saturate. This effect is not understood and requires some further study.

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APPENDIX: STRESS INHOMOGENEITIES IN TRANSVERSE PIEZORESISTANCE MEASUREMENTS

Smith[30] performed transverse piezoresistance measurements by stressing a long sample along its axis and measuring the resistance between large area electrodes deposited on two opposite narrow sides of the sample. These electrodes were sufficiently far away from the sample ends to assure a homogeneous stress distribution between the electrodes. This method suffers from the presence of contact resistances and capacitor-type edge effects which cause the current to be not strictly perpendicular to the stress near the ends of the electrodes.

Being unable to overcome these difficulties we applied the stress along one of the smaller dimensions of an elongated rectangular bar and measured the resistivity with the usual four-probe arrangement. In this case, however, part of the current is flowing near the upper and lower surfaces of the bar over which the

[28] Y. Furukawa, J. Phys. Soc. Japan **15**, 730 (1960).
[29] P. Csavinszky, J. Phys. Soc. Japan **16**, 1865 (1961).
[30] C. S. Smith, Phys. Rev. **94**, 42 (1954).